The Pennsylvania State University

Department of Economics

Econ 390, Section 101, Summer 2007

Midterm Examination #1 - Answer Key .

Problem 1 - Short questions (10 points - 1 point per question)

Questions $1-5$ are based on the following information:

$S=[G_1,G_2,G_3,G_4]$, $A=[G_1,G_2]$, $B=[G_2,G_3]$

1) Find $\bar{A}$

$$\bar{A}=[G_3,G_4]$$

2) Find $\bar{B}$

$$\bar{B}=[G_1,G_4]$$

3) Find $A\cup B$. Are events A and B collectively exhaustive?

$$A\cup B=[G_1,G_2,G_3]$$

$A\cup B\neq S$, hence A and B are NOT collectively exhaustive

4) Find $A\cap B$

$$A\cap B=G_2$$

5) Find $\bar{A}\cap\bar{B}$

$$\bar{A}\cap\bar{B}=G_4$$

Questions $6-8$ are based on the following information:

$E(X)=\mu_X=1$, $E(Y)=\mu_Y=1$, $Var(X)=\sigma_X^2=4$, $Var(Y)=\sigma_Y^2=9$

6) Calculate $E(2X+3Y)$

$$E(2X+3Y)=2E(X)+3E(Y)=2+3=5$$

7) Calculate $Var(3 + 4X)$

$$Var(3 + 4X) = Var(4X) = 16 * Var(X) = 16 * 4 = 64$$

8) Calculate $Var(2 - Y)$

$$Var(2 - Y) = Var(Y) = 9$$

Question 9 is based on the following information:

$P(A|B) = 0.90$, $P(A) = 0.40$

9) Are events A and B statistically independent?

No, since $P(A|B) \neq P(A)$

Question 10 is based on the following information:

$P(A \cap B) = 0.18$, $P(A) = 0.6$ and $P(B) = 0.3$

10) Are events A and B statistically independent?

Yes, $P(A \cap B) = P(A) * P(B) = 0.18$

Problem 2 (15 points)

"Computers Forever Inc." assembles personal computers from parts it purchases on the ebay. Due to defects in some parts 10% of all computers are not working correctly immediately after assembly. To avoid deliveries of computers dead on arrival to its customers firm introduces quality control. Company hires two specialists who carry out quality control procedures. However, sometimes they make mistakes.

If an assembled computer is broken, they will correctly mark it as broken with a probability of 0.85.

If an assembled computer is working, quality control specialists will mark it as working with a probability of 0.95.

1) Find the probability that a computer marked as working is OK when it arrives to a customer

2) Find the probability that a computer is working correctly when quality control marks it as broken.

We know that:

P(Test says "broken" | PC is broken)=0.85

P(Test says "OK" | PC is broken)=0.15

P(Test says "broken"| PC is working)=0.05

P(Test says "OK" | PC is working)=0.95

P(PC is working)=0.90

P(PC is broken)=0.10

Thus we can find:

P(Test says "broken" ∩ PC is broken)=P(Test says "broken" | PC is broken)*P(PC is broken)=0.85*0.10=0.085

P(Test says "OK" ∩ PC is broken)=P(Test says "OK" | PC is broken)*P(PC is broken)=0.15*0.10=0.015

P(Test says "broken"∩ PC is working)=P(Test says "broken"| PC is working)*P(PC is working)=0.05*0.90=0.045

P(Test says "OK" ∩ PC is working)=P(Test says "OK" | PC is working)*P(PC is working)=0.95*0.90=0.855

We can summarize it in the table

	PC is working	PC is broken	
Test says "OK"	0.855	0.015	0.87
Test says "broken"	0.045	0.085	0.13
	0.90	0.10	1

Using Bayes' theorem:

1) P(PC is working | Test says "OK")=P(Test says "OK" ∩ PC is working)/P(Test says "Ok")=0.855/0.87=0.9828

2) P(PC is working | Test says "broken")=P(Test says "broken"∩ PC is working)/P(Test says "broken")=0.045/0.13=0.3462

Problem 3 (15 points)

Computer store has 3 Acer and 5 Dell laptops. Kate purchases 4 laptops for her firm. She is not familiar with computer equipment and just randomly picks 4 laptops from the shelf.

1) What is the probability that she will pick exactly 2 Acers and 2 Dell laptops?

Total number of ways to chose 4 computers out of 8 available is C_4^8

Total number of ways to pick 2 Acers out of the three available is C_2^3

Total number of ways to choose 2 Dells out of the 5 available is C_2^5

P(2 Acers and 2 Dell laptops)=$C_2^3 C_2^5 / C_4^8 = (3 * 10)/70 = 0.4286$

2) What is the probability that all 4 computers are Dell?

Total number of ways to chose 4 computers out of 8 available is C_4^8

Total number of ways to chose 4 Dells out of the 5 available is C_4^5

Total number of ways to pick 0 Acers out of the three available is C_0^3

P(all 4 computers are Dell)=$C_4^5 C_0^3 / C_4^8 = (5 * 1)/70 = 0.0714$

3) What is the probability that all 4 computers are Acer?

0 - there are only 3 Acers, so there is no way to pick 4 Acers.

Problem 4 (20 points)

Adam is an owner of a small business. He produces mouse pads. After 10 years in business, he decides to retire and to sell his enterprise. To find a buyer he prepares a report and summarizes information on the firm sales in a table:

# boxes sold per day	probability	CDF	$x * p(x)$	$x^2 p(x)$
10	0.10	0.10	1.00	10
15	0.20	0.30	3.00	45
20	0.30	0.60	6.00	120
25	0.25	0.85	6.25	156.25
30	0.15	1.00	4.50	135
			20.75	466.25

One box contains 100 pads and market price is 10 dollars per pad.

1) Draw Cumulative Distribution Function (CDF)

2) Calculate the mean number of boxes sold per day

$$\mu_X = \sum_x x * p(x) = 20.75$$

3) Calculate standard deviation of the number of boxes sold per day

$$\sigma = \sqrt{\sum_x x^2 * p(x) - \mu_X^2} = \sqrt{466.25 - 20.75^2} = \sqrt{35.69} \approx 5.97$$

4) Calculate mean daily sales in dollars

Price of one box is $100 * 10 = 1000$ dollars. Company sells 20.75 boxes on average. Hence, mean sales in dollars are just $20.75 * 1000 = 20,750$ dollars

5) Find standard deviation of daily sales in dollars.

Standard deviation in dollars=Standard deviation in boxes*Price of a box= $5.97 * 1000 = 5970$ dollars.

Problem 5 (15points)

A firm purchases 5 expensive laptops for its partners. From previous experience IT specialist know that the probability that each computer dies (due to defects or accidents) during the next 3 years is 0.40.

1) Find the probability that firm a will have to replace all 5 computers in the next 3 years.

Use binomial distribution. $n = 5$, $x = 5$, $p = 0.40$:

$$P(5) = C_5^5 0.40^5 (1 - 0.40)^0 = 0.0102$$

2) Find the probability that firm a will need to replace exactly 3 computers in the next 3 years.

$$P(3) = C_3^5 0.40^3 (1 - 0.40)^2 = 0.2304$$

3) After grading part 2 of the problem, Ivan found out that Kate decided to use binomial distribution to answer part 2. Unlike Kate Bob used Poisson approximation. Was Bob right using Poisson approximation? Evaluate an error he makes using this approximation.

Suppose Bob uses approximation. $\lambda = np = 2 < 7$

$$P(3) = \frac{e^{-2}2^3}{3!} = 0.1804$$

Hence, $error = 0.2304 - 0.1804 = 0.05$. The error is quite significant. Bob is NOT right, $n = 5$ is quite small and $p = 0.4$ is quite high to use approximation.

Hint: Error = Probability obtained by Kate - Probability obtained by Bob.

Problem 6 (15 points)

Local product store sells milk. Sales (in the number of packages) are normally distributed with mean 400 and variance 900. Package price is 1 dollar and 20 cents.

1) Calculate mean revenue from milk sales and its standard deviation.

Denote by $D - revenue$ in dollars, and by P - number of packages sold.

$$E(D) = E(P * 1.2) = 1.2 * E(P) = 1.2 * 400 = 480$$

$$Var(D) = \sigma_D^2 = Var(1.2 * P) = 1.2^2 * Var(P) = 1.2^2 * 900$$

$$\sigma_D = 1.2 * 30 = 36$$

2) Find the probability that revenue from milk sales exceeds 500.

$$P(D > 500) = P(\frac{D - 480}{36} > \frac{500 - 480}{36}) = P(z > 0.55)$$

$$Z\tilde{}N(0, 1)$$

Hence,

$$P(z > 0.55) = 1 - P(z < 0.55) = 1 - F(0.56) = 1 - (0.5 + 0.2123) = 0.2877$$

3) Find the probability that revenue from milk sales is more than 400 but less than 500.

$$P(400 \le D \le 500) = P\left(\frac{400 - 480}{36} \le \frac{D - 480}{36} \le \frac{500 - 480}{36}\right) =$$

$$= P(-2.22 \le Z \le 0.56) = F(0.56) - F(-2.22) = F(0.56) - (1 - F(2.22)) =$$

$$= 0.5 + 0.2123 - (1 - 0.5 - 0.4868) = 0.6991$$

Problem 7 (10 points)

Usually 70% of graduate students in economics choose Game theory as an elective course; 80% choose math for the economists and 60% choose both.

P(Game Theory)= 0.70

P(Math)= 0.80

P(Game Theory ∩ Math)= 0.60

1) Are events "chose Game theory" and "chose math for the economists" statistically independent?

No, since

$$P(Game\ Theory) * P(Math) = 0.70 * 0.80 = 0.56 \neq 0.60 = P(Game\ Theory \cap Math)$$

2) What is the probability that a student chooses at least one of these courses?

$$P(Game\ Theory \cup Math) = P(Game\ Thoery) + P(Math) - P(Game\ Theory \cap Math) = 0.70 + 0.80 - 0.60 = 0.90$$

3) Suppose Alex has already chosen to take math for the economists, what is the probability that he will also take Game theory?

$$P(Game\ Theory \mid Math) = \frac{P(Game\ Theory \cap Math)}{P(Math)} = \frac{0.60}{0.80} = 0.75$$

Problem 8 - Bonus question (15 points)

An insurance company holds fraud insurance policy on 6000 firms. In any given year the probability that any single policy will result in a claim is 0.001. Find the probability that at least tree claims are made in a given year.

To solve this problem we will use Poisson approximation of the binomial distribution. We can do this since n=6000 is large and p=0.001 is small. At the same time $np = 6 < 7$.

Let $\lambda = np = 6$

$$P(X < 3) = P(0) + P(1) + P(2) = \frac{e^{-6}6^0}{0!} + \frac{e^{-6}6^1}{1!} + \frac{e^{-6}6^2}{2!} =$$

$$= 0.0025 + 0.0149 + 0.0446 = 0.0620$$

Hence,

$$P(X \geqslant 3) = 1 - P(X < 3) = 1 - 0.0620 = 0.9380$$