The Pennsylvania State University

Department of Economics

Econ 390, Section 101, Summer 2007

Homework Assignment # 6

Due: Friday, June 29, AT THE BEGINNING OF CLASS

Kate works as a senior consultant for one of the largest real estate companies in NY. She asks her assistant to figure out the relationship between apartment's price and distance between the apartment and Central Park. To do this, she provides data on recent transactions of the firm. All apartments in the sample are approximately of the same footage and very similar in other characteristics except distance to the Central Park. Overall she gives 15 data points to her assistant:

y_i	x_i	$(y_i-\bar{Y})^2$	$(x_i-\bar{X})^2$	$(y_i-\bar{Y})(x_i-\bar{X})$	$\hat{y}_i$	e_i	e_i^2	x_i^2
3.20	0.50	0.20	1.70	−0.59	3.28	−0.08	0.01	0.25
3.10	0.70	0.12	1.22	−0.39	3.20	−0.10	0.01	0.49
3.25	0.80	0.25	1.01	−0.50	3.16	0.09	0.01	0.64
3.00	1.00	0.06	0.65	−0.20	3.08	−0.08	0.01	1.00
3.00	1.50	0.06	0.09	−0.08	2.87	0.13	0.02	2.25
2.90	1.60	0.02	0.04	−0.03	2.83	0.07	0.00	2.56
3.15	1.70	0.16	0.01	−0.04	2.79	0.36	0.13	2.89
2.80	1.80	0.00	0.00	0.00	2.75	0.05	0.00	3.24
2.70	1.80	0.00	0.00	0.00	2.75	−0.05	0.00	3.24
2.60	1.90	0.02	0.01	−0.01	2.71	−0.11	0.01	3.61
2.45	2.00	0.09	0.04	−0.06	2.67	−0.22	0.05	4.00
2.43	2.50	0.10	0.49	−0.22	2.46	−0.03	0.00	6.25
2.15	3.00	0.36	1.43	−0.72	2.26	−0.11	0.01	9.00
2.30	3.10	0.20	1.68	−0.58	2.22	0.08	0.01	9.61
2.19	3.15	0.31	1.81	−0.75	2.20	−0.01	0.00	9.92
$\frac{\sum_{i=1}^n y_i}{n}$	$\frac{\sum_{i=1}^n x_i}{n}$	$\frac{\sum_{i=1}^n (y_i-\bar{Y})^2}{n-1}$	$\frac{\sum_{i=1}^n (x_i-\bar{X})^2}{n-1}$	$\frac{\sum_{i=1}^n (x_i-\bar{X})(y_i-\bar{Y})}{n-1}$		$\sum e_i$	$\sum e_i^2$	$\sum x_i^2$
$\frac{41.25}{15}=2.7480$	$\frac{27.05}{15}=1.8033$	$\frac{1.9784}{14}=0.1413$	$\frac{10.1723}{14}=0.7266$	$\frac{-4.17}{14}=-0.2982$		0.0000	0.2650	58.9525

1. Calculate $\bar{X}$ and $\bar{Y}$

$$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}x_i=1.8033$$

$$\bar{Y}=\frac{1}{n}\sum_{i=1}^{n}y_i=2.7480$$

(See columns 1 and 2 of the table)

2. Calculate S_X and S_Y

$$S_X = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})^2} = \sqrt{0.7266} \approx 0.8524$$

$$S_Y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{Y})^2} = \sqrt{0.1413} \approx 0.3759$$

3. Calculate covariance between X and Y: $Cov(X, Y)$

$$Cov(X, Y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{X})(y_i - \bar{Y}) = -0.2982$$

4. Assume that price is a linear function of distance: $y_i = b_0 + b_1 * X_i$. Estimate b_0 and b_1 using expressions we have discussed in class.

$$b_1 = \frac{Cov(X, Y)}{S_X^2} = \frac{-0.2982}{0.7266} \approx -0.4104$$

$$b_0 = \bar{Y} - b_1 * \bar{X} = 2.7480 - (-0.4104) * 1.8033 \approx 3.4881$$

5. How can you interpret b_1?

On average, keeping all other things constant, increase in distance to Central Park by 1 mile cause price of appartment to decrease by 0.41 million dollars.

6. Calculate R^2. What does it tell you?

We calculate $\hat{y}_i$ first:

$$\hat{y}_i = 3.4881 - 0.4104 * x_i$$

Then we can obtain values of residuals: $e_i = y_i - \hat{y}_i$, and take their square: $e_i^2 = (y_i - \hat{y}_i)^2$

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n}(y_i - \bar{Y})^2} = 1 - \frac{0.2650}{1.9784} \approx 0.8661$$

Since $\sum_{i=1}^{n} e_i^2$, is just the sum of column 8, while $\sum_{i=1}^{n}(y_i - \bar{Y})^2$ is the sum of column 3.

7. Calculate the variance of b_0 and the variance of b_1

$$S_e^2 = \frac{1}{n-2}\sum_{i=1}^{n} e_i^2 = \frac{0.2650}{13} \approx 0.02038$$

$$\hat{V}(\hat{b}_1) = \frac{S_e^2}{\sum_{i=1}^{n}(x_i - \bar{X})^2} = \frac{0.0189}{10.1723} \approx 0.0020$$

$$\hat{S}_{\hat{b}_1} = \sqrt{\hat{V}(\hat{b}_1)} = \sqrt{0.0020} \approx 0.0448$$

$$\hat{V}(\hat{b}_0) = \frac{S_e^2 \sum_{i=1}^{n} x_i^2}{n\sum_{i=1}^{n}(x_i - \bar{X})^2} = \frac{0.02038 * 58.9525}{15 * 10.1723} \approx 0.007875$$

$$\hat{S}_{\hat{b}_0} = \sqrt{\hat{V}(\hat{b}_0)} = \sqrt{0.007875} \approx 0.08874$$

8. Calculate $t-$statistics for b_1. Can you reject $H_0 : b_1 = 0$ in favor of $H_1 : b_1 \neq 0$ at 5% significance level ?

$$t = \frac{\hat{b}_1}{\hat{S}_{\hat{b}_1}} = \frac{-0.4104}{0.0448} \approx -9.17$$

$$t_{0.025,13} = 2.160$$

$$t = -9.17 < -t_{0.025,13} = 2.160$$

Hence, we can reject H_0 at 5% level.