The Pennsylvania State University

Department of Economics

Econ 390, Section 101, Summer 2007

Homework Assignment # 5

Solution

Problem 1

A random sample of size $n = 25$ is obtained from a population with variance σ^2, and the sample mean is computed. Test the null hypothesis $H_0 : \mu = 100$ versus the alternative hypothesis $H_1 : \mu > 100$ with $\alpha = 0.05$. Compute the critical value $\bar{X}_c$, and state your decision rule for the following options:

a. The population variance is $\sigma^2 = 225$

b. The population variance is $\sigma^2 = 900$

c. The population variance is $\sigma^2 = 400$

For a, b and c we are going to use the following rule:

Reject H_0 if $\bar{x} > \bar{x}_c = \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}}$, $Z_{0.05} = 1.645$

a. $\bar{x}_c = \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 100 + 1.645 \frac{\sqrt{225}}{\sqrt{25}} = 100 + 1.645 \frac{15}{5} \approx 104.94$

b. $\bar{x}_c = \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 100 + 1.645 \frac{\sqrt{900}}{\sqrt{25}} = 100 + 1.645 \frac{30}{5} \approx 109.87$

c. $\bar{x}_c = \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} = 100 + 1.645 \frac{\sqrt{400}}{\sqrt{25}} = 100 + 1.645 \frac{20}{5} \approx 106.58$

Problem 2

A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution with standard deviation of 0.4%. A random sample of 64 pills from a production run was checked, and the sample mean impurity concentration was found to be 3.07%.

a. Test at the 5% level the null hypothesis that the population mean impurity concentration is 3% against the alternative that it is more than 3%.

b. Find p-value for this test

a. $H_0 : \mu = 3$, $H_1 : \mu > 3$. We are going to reject H_0 if $Z > Z_{0.05} = 1.645$

$$Z = \frac{3.07 - 3}{0.4/\sqrt{64}} = 1.4$$

Since $1.4 < 1.645$ we can not reject H_0 at 5% level

b.

$$P(\bar{X} > 3.07 | H0 : \mu = 3) = P(\frac{\bar{X} - 3}{0.4/\sqrt{64}} > \frac{3.07 - 3}{0.4/\sqrt{64}} | H0 : \mu = 3) = P(Z > 1.4 | H0 : \mu = 3) =$$

$$= 1 - F(1.4) = 1 - 0.9192 \approx 0.0808$$

Problem 3

A random sample of 172 marketing students was asked to rate from 1 (not important) to 5 (extremely important) health benefits as a job characteristic. The sample mean rating was 3.31, and the sample deviation was 0.70. Test at the 1% significance level the null hypothesis that the population mean rating is at most 3.0 against the alternative that it is bigger than 3.0.

We are testing $H_0 : \mu \leq 3$ against: $H_1 : \mu > 3$.

From the problem we know that: $S = 0.70$ and $\bar{X} = 3.31$

$$t = \frac{3.31 - 3}{0.7/\sqrt{172}} = 5.81$$

$$t_{171,0.01} = 2.326$$

(While using tables for student distribution take value for ∞ number of degrees of freedom)

Since $5.81 > 2.326$ we can reject H_0 at 1% level.

Problem 4

In a random sample of 998 adults in the United States, 17.3% of the sample members indicated some measure of disagreement with this statement "Globalization is more than an economic trade system - instead in includes institutions and culture". Test at the 5% level the hypothesis that at least 25% of all U.S. adults would disagree with this statement.

$H_0 : P \geq .25$, $H_1 : P < 0.25$. We can reject H_0 if $Z < -Z_\alpha$.

$$Z_{0.05} = 1.645$$

$$Z = \frac{0.173 - 0.25}{\sqrt{0.25 * 0.75/998}} = -5.62$$

Since $-5.62 < -1.645$, we can reject H_0 at 5% level.

Problem 5.

A company that receives shipments of batteries test a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with standard deviation 3 hours. For one particular shipment the mean lifetime for a sample of nine batteries was 48.2 hours.

a. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.

b. Find the power of a 10%−level test when the true mean lifetime of batteries is 49 hours.

a. $H_0 : \mu \geq 50$, $H_1 : \mu < 50$,

$$Z = \frac{48.2 - 50}{3/\sqrt{9}} = \frac{-1.8}{1} = -1.8$$

$Z_{0.10} = 1.28$

Since, $Z < Z_{0.10}$, we have to reject H_0 in favor of H_1

b. $\bar{X}_C = \mu_0 - Z_\alpha \frac{\sigma}{\sqrt{n}} = 50 - 1.28 \frac{3}{\sqrt{9}} = 50 - 1.28 = 48.72$

$$\beta = P(\bar{X} > \bar{X}_C | \mu^* = 49) = P\left(\frac{\bar{X} - \mu^*}{\sigma/\sqrt{n}} > \frac{\bar{X}_C - \mu^*}{\sigma/\sqrt{n}} \mid \mu^* = 49\right) =$$

$$= P\left(Z > \frac{\bar{X}_C - \mu^*}{\sigma/\sqrt{n}} \mid \mu^* = 49\right) = 1 - P\left(Z \leq \frac{\bar{X}_C - \mu^*}{\sigma/\sqrt{n}} \mid \mu^* = 49\right) =$$

$$= 1 - F\left(\frac{\bar{X}_C - \mu^*}{\sigma/\sqrt{n}}\right) = 1 - F\left(\frac{48.72 - 49}{1}\right) = F\left(0.28\right) \approx 0.61$$

Power of the test for $\mu^* = 49$ is equal to $1 - \beta = 1 - 0.61 = 0.39$