The Pennsylvania State University

Department of Economics

Econ 390, Section 001, Summer 2007

Homework Assignment #1

SOLUTION

Problem 1 (Warm up)

$$S = [E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10}]$$

1. Given $A = [E_2, E_4, E_7, E_9]$, define $\bar{A}$

$$\bar{A} = [E_1, E_3, E_5, E_6, E_8, E_{10}]$$

2. Additionally given that $B = [E_1, E_3, E_4, E_7, E_8, E_9]$

a) What is intersection of A and B

$$A \cap B = [E_4, E_7, E_9]$$

b) What is union of A and B

$$A \cup B = [E_1, E_2, E_3, E_4, E_7, E_8, E_9]$$

c) Is the union of A and B collectively exhaustive?

No - it does not contain all the elements that S does: $A \cup B \neq S$

d) What is intersection of $\bar{A}$ and $\bar{B}$

$$\bar{A} = [E_1, E_3, E_5, E_6, E_8, E_{10}]$$

$$\bar{B} = [E_2, E_5, E_6, E_{10}]$$

$$\bar{A} \cap \bar{B} = [E_5, E_6, E_{10}]$$

Problem 2

Bill operates a small used car lot in the middle of nowhere in Washington, DC. He has three Mercedes (M_1, M_2, M_3) and two Fords (F_1, F_2). One day, two customers, Hillary and Monica, come to his lot and each selects a car. The customers do not know each other, and there is no communication between them. Let the events A and B be defined as follows:

$A =$ The customers select at least one Ford

$B =$ The customers select two cars of the same model.

1. Describe event A

$$A = \{[F_1, F_2], [M_1, F_2], [M_2, F_2], [M_3, F_2], [F_1, M_1], [F_1, M_2], [F_1, M_3]\}$$

2. Describe event B

$$B = \{[F_1, F_2], [M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

3. Describe the complement of A

   Both customers bought Mercedes - no Fords!:

$$\bar{A} = \{[M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

4. Show that $(A \cap B) \cup (\bar{A} \cap B) = B$

$$(A \cap B) = \{[F_1, F_2]\}$$

$$(\bar{A} \cap B) = \{[M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

$$\{[F_1, F_2]\} \cup \{[M_1, M_2], [M_1, M_3], [M_2, M_3]\} = B$$

5. Show that $A \cup (\bar{A} \cap B) = (A \cup B)$

$$(\bar{A} \cap B) = \{[M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

$$A \cup (\bar{A} \cap B) =$$

$$\{[F_1, F_2], [M_1, F_2], [M_2, F_2], [M_3, F_2], [F_1, M_1], [F_1, M_2], [F_1, M_3], [M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

$$(A \cup B) =$$

$$\{[F_1, F_2], [M_1, F_2], [M_2, F_2], [M_3, F_2], [F_1, M_1], [F_1, M_2], [F_1, M_3], [M_1, M_2], [M_1, M_3], [M_2, M_3]\}$$

Problem 3

A 7 person board of directors to be chosen at random from a group of 12 candidates. Five of the candidates have Ph.D. in Business or in Economics and the rest are lawyers.

1. Find the probability that committee has exactly 4 lawyers.

The total number of ways to choose 7 persons out of 12 candidates is:

$$C_7^{12} = \frac{12!}{7!(12-7)!} = \frac{12!}{7!5!} = 792$$

The number of ways it is possible to choose 4 lawyers 7 is:

$$C_4^7 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!} = 35$$

The number of ways it is possible to choose 3 Ph.D. out of 5:

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = 10$$

Probability that committee has exactly 4 lawyers is

$$p(exactly\ 4\ lawyers) = \frac{C_4^7 * C_3^5}{C_7^{12}} = \frac{35 * 10}{792} \approx 0.44$$

2. Find the probability that there is at least 1 director with Ph.D. in Business or in Economics.

(Hint: What is the complement of the event "there is at least 1 director with Ph.D. in Business or in Economics")

Complement events is "No directors with Ph.D. in Business or in Economics", i.e. all are lawyers

The total number of ways to choose 7 persons out of 12 candidates is:

$$C_7^{12} = \frac{12!}{7!(12-7)!} = \frac{12!}{7!5!} = 792$$

The total number of ways it is possible to choose 7 lawyers from 7 available is... 1

The total number of ways it is possible to choose 0 Ph.D.'s out of 5 is:

$$C_0^5 = \frac{5!}{5!0!} = 1$$

Probability that "there is at least 1 director with Ph.D. in Business or in Economics" is just

$$1 - \frac{1 * 1}{792} = \frac{791}{792} \approx 0.99$$

Problem 4

Alex is a car dealer somewhere in Siberia, Russia. From previous sales experience he knows that about 79% of his clients require him to install additional heater in their new car; 85% ask for additional side airbags; and, 69% will not buy a car until both heater and airbags are installed.

1. What is the probability that customer will ask for at least one of these two systems?

Define A ="client requires to install additional heater"

B ="client ask for additional side airbags"

$P(A) = 0.79$, $P(B) = 0.85$ and $P(A \cap B) = 0.69$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.79 + 0.85 - 0.69 = 0.95$$

2. Suppose Kate asks for the heater system. What is the probability that she will also ask to install additional airbags?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.69}{0.79} \approx 0.87$$

3. Suppose Michael requires to add airbags. What is the probability that he will also ask to install additional heater?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.69}{0.85} \approx 0.81$$

Problem 5

A publisher sends advertising materials for an accounting text to 80% of all professors teaching the appropriate accounting course. Thirty percent of the professors who received this material adopted the book, as did 10% of the professors who did not receive the material. What is the probability that a professor who adopts the book has received the advertising material?

$A_1 =$"Professor receives advertising material"

$A_2 =$"Professor does not receive advertising material"

$B_1 =$"Adopts the book"

$B_2 =$"Does not adopt the book"

$P(A_1) = 0.8$, $P(A_2) = 1 - 0.8 = 0.2$, $P(B_1|A_1) = 0.3$, $P(B_1|A_2) = 0.1$

$$P(A_1|B_1) = \frac{P(B_1|A_1)P(A_1)}{P(B_1|A_1)P(A_1) + P(B_1|A_2)P(A_2)} = \frac{0.3 * 0.8}{0.3 * 0.8 + 0.1 * 0.2} = 0.923$$

Problem 6

An insurance company estimated that 30% of all automobile accidents were partly caused by weather conditions and that 20% of all automobile accidents involved bodily injury. Further, if the accident involved bodily injury, then in 40% of the cases it was partly caused by weather conditions.

1. What is the probability that a randomly chosen accident both was partly caused by weather conditions and involved bodily injury?

$A =$"accident was partly caused by weather conditions"

$B =$"accident involved bodily injury"

$P(A) = 0.30$, $P(B) = 0.20$, $P(A|B) = 0.40$

$$P(A \cap B) = P(A|B)P(B) = 0.40 * 0.20 = 0.08$$

2. Are the events "Partly caused by weather conditions" and "Involved bodily injury" independent?

No:

$$P(A)P(B) = 0.06 \neq P(A \cap B) = 0.08$$

3. If a randomly chosen accident was partly caused by weather conditions, what is the probability that it involved bodily injury?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.08}{0.30} \approx 0.27$$

4. What is the probability that a randomly chosen accident both was not partly caused by weather conditions and did not involve bodily injury?

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 1 - 0.30 - 0.20 + 0.08 = 0.58$$

Problem 7

A survey carried out for a supermarket classified customers according to whether their visits to the store are frequent or infrequent and whether they often, sometimes or never purchase generic products. The accompanying table gives the proportions of people surveyed in each of the six joint classifications:

	Purchase of Generic products		
	Often	Sometimes	Never
Frequent visitor	0.12	0.48	0.19
Infrequent visitor	0.07	0.06	0.08

1. What is the probability that a customer both is a frequent shopper and often purchase generic products?

$$P(F \cap O) = 0.12$$

2. What is the probability that a customer who never buys generic products visits the store frequently?

$$P(F|N) = \frac{P(F \cap N)}{P(N)} = \frac{0.19}{0.19 + 0.08} \approx 0.704$$

3. Are the events "Never buys generic products" and "Visits the store frequently" independent?

$P(N) = 0.19 + 0.08 = 0.27,\ P(F) = 0.12 + 0.48 + 0.19 = 0.79,\ P(N) * P(F) = 0.27 * 0.79 = 0.2133$

No, since:

$$P(N) * P(F) = 0.2133 \neq 0.19 = P(N \cap F)$$

4. What is the probability that a customer who infrequently visits the store often buys generic products?

$$P(O|I) = \frac{P(O \cap I)}{P(I)} = \frac{0.07}{(0.07 + 0.06 + 0.08)} = \frac{0.07}{0.21} \approx 0.33$$

5. Are the events "Often buys generic products" and "Visits the store infrequently" independent?

$P(O) = 0.12 + 0.07 = 0.19,\ P(I) = 0.21.$

No, since:

$$P(O)P(I) = 0.0399 \neq 0.07 = P(O \cap I)$$